Blakers Mathematics Competition 2023

Open to first to third year students of any Western Australian university, with prizes sponsored by the UWA Mathematics Union.

The Competition begins **Friday, 21 July** and ends **Friday, 15 September** 2023.

You may use any source of information except other people. In particular, you may use computer packages (though contributed solutions are expected to have complete mathematical proofs). Extensions and generalisations of any problem are invited and are taken into account when assessing solutions. Also, elegance of solution is particularly favoured and desired. We reserve the right to not read a solution if it is "messy"; we expect neat solutions (so perhaps avoid submitting a first draft).

We are delighted to announce that prizes will once again be awarded, with their being supplied by new sponsor: the **UWA Mathematics Union**.

Solutions are to be emailed as PDF attachments to Greg Gamble (`greg.gamble@uwa.edu.au`) **by 4 pm on Friday, 15 September.**

Instructions for solutions: Include the following information in the body of your email:

name,
student ID number,
home address (optional),
e-mail address,
university where enrolled,
number of years you have been attending any tertiary institution, and
list of the questions completed and attached as PDFs to your email.

Please scan each question to a separate PDF file and name the file according to the protocol: $\langle$`YourLastName`$\rangle\langle n\rangle$`.pdf` where $\langle n\rangle$ is the number of the question.

Before you scan, please ensure you have started each new problem on a new page, and that at the top of each page is the number of the page, and your full name.

Note. Our convention is that $\mathbb{N} = \{1, 2, \dots\}$ (the positive integers).

2023 Problems

1. Quad warm-up exercise

Find the area of the quadilateral of largest area with sides 4, 16, 17, 23.

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2. Geometrically aligned

Let X, Y be points on sides LM, MK of acute triangle KLM, respectively, and let H be the orthocentre of $\triangle KLM$.
Prove that the points of intersection of the circles with diameters KX and LY, and H are collinear.

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3. Cyclic divisibility

Let $x, y, z \in \mathbb{N}$ such that x divides y^4, y divides z^4, and z divides x^4.
Prove xyz divides $(x + y + z)^{23}$.

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4. Squared circle?

Is it possible to place 2023 consecutive natural numbers around a circle so that the product of each adjacent pair is a perfect square?

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5. Nonnegatively polynomial

Suppose $p(x)$ is a polynomial over $\mathbb{R}$ such that

$$p(x) - p'(x) - p''(x) + p'''(x) \geqslant 0, \text{ for all } x \in \mathbb{R}.$$

Prove $p(x) \geqslant 0$ for all $x \in \mathbb{R}$.
