Blakers Mathematics Competition 2024

Open to first to third year students of any Western Australian university,
with prizes sponsored by the UWA Mathematics Union.

The Competition begins **Monday, 16 September** and ends **Friday, 18 October** 2024.

You may use any source of information except other people. In particular, you may use computer packages (though contributed solutions are expected to have complete mathematical proofs). Extensions and generalisations of any problem are invited and are taken into account when assessing solutions. Also, elegance of solution is particularly favoured and desired. We reserve the right to not read a solution if it is "messy"; we expect neat solutions (so perhaps avoid submitting a first draft).

We are delighted to announce that prizes will once again be awarded, with their being supplied by new sponsor: the **UWA Mathematics Union**.

Solutions are to be emailed as PDF attachments to Greg Gamble (`greg.gamble@uwa.edu.au`) **by 4 pm on Friday, 18 October.**

Instructions for solutions: Include the following information in the body of your email:

name,
student ID number,
home address (optional),
e-mail address,
university where enrolled,
number of years you have been attending any tertiary institution, and
list of the questions completed and attached as PDFs to your email.

Please scan each question to a separate PDF file and name the file according to the protocol: ⟨`YourLastName`⟩⟨n⟩`.pdf` where ⟨n⟩ is the number of the question.

Before you scan, please ensure you have started each new problem on a new page, and that at the top of each page is the number of the page, and your full name.

Note. Our convention is that $\mathbb{N} = \{1, 2, \dots\}$ (the positive integers).

2024 Problems

1. Radical annual empowered

Does there exist $N \in \mathbb{N}$ such that

$$(\sqrt{2024} - \sqrt{2023})^{2022} = \sqrt{N} - \sqrt{N-1}?$$

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2. Squared triangle

On sides AB, BC and CA of $\triangle ABC$, construct squares ABB_1A_1, BCC_1B_2, CAA_2C_2, respectively.
Let P be the centre of square BCC_1B_2.
Prove lines A_1C, A_2B an AP concur.
Note. Lines *concur* if they pass through a common point.

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3. Fractional sum

Show that

$$\frac{2023}{2}-\frac{2022}{3}+\frac{2021}{4}+\cdots-\frac{2}{2023}+\frac{1}{2024}=\frac{1}{1013}+\frac{3}{1014}+\cdots+\frac{2023}{2024}.$$

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4. Radically signed

Somewhat informally, let $a_1=2\pm\sqrt{2}$, $a_{n+1}=2\pm\sqrt{a_n}, n\geqslant 1$.
Let $A(n)$ be the set of all expressions a_n, e.g.

$$A(2)=\left\{2+\sqrt{2+\sqrt{2}},2+\sqrt{2-\sqrt{2}},2-\sqrt{2+\sqrt{2}},2-\sqrt{2-\sqrt{2}}\right\}.$$

(a) Prove that all the elements of $A(n)$ are real.

(b) Compute the product

$$\prod_{a\in A(n)} a.$$

(c) If $A(24)$ is sorted into ascending order, what position is the element whose signs in order are

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5. Integer and fractionally parted

For $x,y\in\mathbb{N}$, let $\overline{x.y}$ be the number whose integer part is composed of the decimal digits of x, and whose fractional part is composed of the decimal digits of y, e.g. if $x=124$ and $y=816$ then $\overline{x.y}=124.816$.
Find all solutions of

$$\frac{a}{b}=\overline{b.a},$$

where a,b are coprime.