UWA Mathematics Union

Blakers Mathematics Competition 2025

Open to first to third year students of any Western Australian university,
with prizes sponsored by the UWA Mathematics Union.

The Competition begins **Monday, 1 September** and ends **Friday, 3 October** 2025.

You may use any source of information except other people. In particular, you may use computer packages (though contributed solutions are expected to have complete mathematical proofs and human creativity). Extensions and generalisations of any problem are invited and are taken into account when assessing solutions. Also, elegance of solution is particularly favoured and desired. We reserve the right to not read a solution if it is "messy"; we expect neat solutions (so perhaps avoid submitting a first draft).

We are delighted to announce that prizes will once again be awarded, with their being supplied by new sponsor: the **UWA Mathematics Union**.

Solutions are to be emailed as PDF attachments to Greg Gamble (`greg.gamble@uwa.edu.au`) **by 4 pm on Friday, 3 October.**

Instructions for solutions: Include the following information in the body of your email:

name,
student ID number,
home address (optional),
e-mail address,
university where enrolled,
number of years you have been attending any tertiary institution, and
list of the questions completed and attached as PDFs to your email.

Please scan each question to a separate PDF file and name the file according to the protocol: ⟨`YourLastName`⟩⟨n⟩`.pdf` where ⟨n⟩ is the number of the question.

Before you scan, please ensure you have started each new problem on a new page, and that at the top of each page is the number of the page, and your full name.

Note. Our convention is that $\mathbb{N} = \{1, 2, \dots\}$ (the positive integers),
a lightning bolt ($\lightning$) signals a contradiction, and
serial exponentiation is *right-associative*, i.e.

$$a^{b^c} = a^{(b^c)}.$$

2025 Problems

1. Legion amusement

Legion's numbers of the first, second and third kinds are

$$L_1 = 666^{666},$$
$$L_2 = 666!^{666!},$$
$$L_3 = 666^{666^{666}},$$

respectively. Also, denote the sum of the digits of N, by $S(N)$.

(a) Find the sum of the last 666 666 digits of L_2.

(b) Which is larger $S(L_2)$ or 10^{2000}?

(c) Which is larger L_2 or L_3?

(d) Find $S(S(S(S(L_3))))$?

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2. Fourth empowerment

Let $S = \{1, 4, 9, \ldots, k^2, \ldots, 2025\}$ and let T be the subset of S such that no pair of distinct elements of T has a product that is a perfect fourth power.

(a) Find the maximum cardinality of T.

(b) Find the number of such sets T of maximum cardinality.

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3. Hexadecimal tetration

What is the last hexadecimal digit of

$$B^{B^{B^{\cdot^{\cdot^{\cdot^{B}}}}}},$$

where there are B Bs?

Note. Hexadecimal is base sixteen, which has digits $0, 1, 2, \ldots, 9, A, B, C, D, E, F$. You may find it convenient to refer to such an expression as a *tower* of so many Bs, or to a *tetration*. The n^{th} **tetration** of a, written ${}^n a$, means $a^{a^{\cdot^{\cdot^{\cdot^{a}}}}}$ (where there are n copies of a).

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4. Powerfully functional

Let function $f : \mathbb{R} \to \mathbb{R}$ satisfy

$$f(x^3 + y^3) = (x + y)\big(f(x)^2 - f(x)f(y) + f(y)^2\big) \qquad (*)$$

for all $x, y \in \mathbb{R}$.

Prove $f(2025x) = 2025f(x)$.

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5. Common area

Suppose points X and Y lie in that order on side BC of acute triangle ABC, such that $\angle BAX = \angle CAY$.
Points M, N lie on AB, AC, respectively, such that $YM \perp AB$ and $YN \perp AC$.
Line AX produced intersects the circumcircle of ABC again at P.

Prove the areas of triangle ABC and quadrilateral $AMPN$ are equal.